A-Level Physics Revision Notes

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20 January 2017

1 Particles

$$\text{Specific charge} = \frac{\text{charge}}{\text{mass}}$$

In ${}^A_Z X$ notation: A = mass number (protons + neutrons), Z = proton number.
Isotopes — atoms with same proton number but different mass numbers.

1.1 Stable and unstable nuclei

The strong nuclear force overcomes electrostatic repulsion between protons in the nucleus.

- range: 3-4 fm (about diameter of a small nucleus)
  - attractive from 0.5 fm to around 3 fm or 4 fm, repulsive below 0.5 fm.
- same effect between two protons as two neutrons or a neutron & proton.
- exchange particle: π

1.1.1 Radioactive decay

- Alpha α radiation consists of ${}^4_2\alpha$ particles:

  $${}^A_Z X \rightarrow {}^{A-4}_{Z-2} Y + {}^4_2 \alpha$$

- Beta β radiation is fast-moving electrons, hence symbol ${}^{0}_{-1}\beta$ or β^-

  $${}^A_Z X \rightarrow {}^{A}_{Z+1} Y + {}^0_{-1} \beta + \overline{V_e}$$

  When a neutron in the nucleus changes into a proton a β^- particle is released and instantly emitted, along with an electron antineutrino.
  The existence of the neutrino was hypothesised to account for the conservation of energy in β^- decay — it went unproven until antineutrinos were detected.

- Gamma γ radiation is electromagnetic radiation emitted by an unstable nucleus with too much energy following α or β emission. It has no mass and no charge.

1.2 Particles, antiparticles and protons

Every particle has a corresponding antiparticle. When antimatter and matter meet, they destroy each other and radiation is released — annihilation.

$$1 \text{ electron Volt} = 1 \text{ eV} = 1.6 \cdot 10^{-19} \text{ J}$$

- Annihilation — particle and corresponding antiparticle meet and their mass is converted to radiation energy.
  - 2 photons (γ) are produced to ensure a total momentum of zero following the collision

    minimum energy of each photon, $hf_{min} = E_0$ where E_0 is rest energy of particle

- Pair production — a photon γ creates a particle and corresponding antiparticle.

$$\text{minimum energy of photon needed, } hf_{min} = 2E_0 \text{ where } E_0 \text{ is rest energy of particle}$$

- The antiparticle theory states for every particle there is a corresponding antiparticle that:
  - annihilates the particle & itself if they meet — converting their total mass into photons
  - has exactly the same rest mass as the particle
  - has exactly opposite charge to the particle if the particle is charged.
- The electron's antiparticle is the positron (β^+). Positron emission occurs when a proton changes into a neutron in an unstable nucleus with too many protons.

$${}^{A}_{Z}X \rightarrow^{A}_{Z-1} Y +^{0}_{+1} \beta + V_e$$

1.3 Particle interactions

The electromagnetic force between two charged particles or objects is due to the exchange of virtual photons (γ). eg two protons will repel each other.

The weak nuclear force affects only unstable nuclei — it is responsible for neutron → proton (β^-) and proton → neutron (β^+) decay. In both, a particle and antiparticle are created but do not correspond.

- a neutron—neutrino interaction changes the neutron to a proton and results in β^- emission
  - W^- boson exchange particle

$$\mathrm{n} + V_e \rightarrow \mathrm{p} + \beta^-$$

- a proton—antineutrino interaction changes the proton to a neutron and results in β^+ emission.
  - W^+ boson exchange particle

$$\mathrm{p} + \overline{V_e} \rightarrow \mathrm{n} + \beta^+$$

- These interactions are due to the exchange of W bosons. Unlike photons they have:
  - non-zero rest mass
  - very short range $\leq \frac{1}{1000}$ fm
  - positive or negative charge

If no neutrino or antineutrino is present, W^- decays to $\beta^- + \overline{V_e}$, and W^+ decays to $\beta^+ + V_e$. Note that charge is conserved.

- β^- decay: $\mathrm{n} \rightarrow \mathrm{p} + \beta^- + \overline{V_e}$
- β^+ decay: $\mathrm{p} \rightarrow \mathrm{n} + \beta^+ + V_e$

In electron capture a proton in a proton-rich nucleus turns into a neutron through weak force interaction with an inner-shell electron.

$$\mathrm{p} + \mathrm{e}^- \rightarrow \mathrm{n} + V_e$$

The same can happen when a proton & electron collide at very high speed. For an electron with sufficient energy the overall change could occur as W^- exchange from e^- to p.

1.4 Particle classifications

Hadrons are particles/antiparticles which interact through the strong force — protons, neutrons, π-ons and K-ons.

Hadrons can interact through all four interactions. They interact trough the strong force and electromagnetic interaction if charged. Other than the proton, which is stable, hadrons tend to decay through the weak force.

Hadrons are further divided into:

- Baryons — protons & all other hadrons incl. neutrons that decay into protons directly or otherwise
- Mesons — hadrons not including protons in their decay products ie π and K mesons.

Leptons do not interact through the strong force — they interact only through the weak, gravitational and (if charged) electromagnetic interactions.

- Lepton decays
  - K $\rightarrow \pi$, or $\mu + \overline{V_\mu}$, or $\overline{\mu} + V_\mu$
  - $\pi^\pm \rightarrow \mu + \overline{V_\mu}$ or $\overline{\mu} + V_\mu$
  - $\pi^0 \rightarrow \gamma$ (high energy photons)
  - $\mu \rightarrow \mathrm{e}^- + \overline{V_e}$
  - $\overline{\mu} \rightarrow \mathrm{e}^+ + V$
  - Note that decays always obey conservation rules for energy, momentum & charge.

$$\text{rest energy of products} = \text{total energy before} - \text{kinetic energy of products}$$

1.5 Leptons and quarks

Leptons and antileptons can interact to produce hadrons — this is due to the production of quarks during these events.
An up-quark has charge $+\frac{2}{3}$, a down-quark charge $-\frac{1}{3}$

- In a lepton—hadron interaction a neutrino or antineutrino can change into or from a corresponding charged lepton.
  - $V_e + \mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^-$
  - but even though Q conserved $V_e + \mathrm{n} \not\rightarrow \overline{\mathrm{p}} + \mathrm{e}^+$
  - this is because the lepton number must balance
- Muon μ decay: μ changes to V_e and e^- created to conserve charge, and $\overline{V_e}$ to preserve lepton number
  - eg $\mu^- \rightarrow \mathrm{e}^- + \overline{V_e} + V_\mu$
  - But $\mu^- \not\rightarrow \mathrm{e}^- + \overline{V_e} + \overline{V_\mu}$ even though charge conserved—because lepton number is not.
  - Muon can change only into a muon neutrino (not antineutrino).
  - Electron can only be created with an electron antineutrino
- Lepton number +1 for any lepton, -1 for any antilepton, 0 for non-lepton
- From smallest to greatest rest mass: e^- ...×200... μ^-, $\pi^{0/\pm}$, $\mathrm{K}^{0/\pm}$ ... p
  - K $\rightarrow \pi$, $\mu^- + \overline{V_\mu}$, or $\mu^+ + V$
  - $\pi^\pm \rightarrow \mu^+\overline{V_\mu}$, or $\mu^- + V_\mu$
  - $\pi^0 \rightarrow \gamma$ (high energy photons)
  - $\mu \rightarrow \mathrm{e}^- + \overline{V\mathrm{e}}$
  - $\overline{\mu} \rightarrow \mathrm{e}^+ + V_e$

1.5.1 Strangeness

Strange particles are produced through the strong interaction and decay through the weak interaction.

- For strangeness +1, need antistrange quark. For strangeness -1, need strange quark.
- Mesons are hadrons consisting of two quarks—one and antiquark.
  - π^0 = any q—$\overline{\mathrm{q}}$ combination — so can be strange
    - $\pi^+ = \mathrm{u}\overline{\mathrm{d}}$, $\pi^- = \overline{\mathrm{u}}\mathrm{d}$
  - each pair of charged mesons is a particle-antiparticle pair
  - antiparticle of any meson is a q—$\overline{\mathrm{q}}$ pair thus another meson.
  - hence only K are strange
    - $\mathrm{K}^0 = \mathrm{d}\overline{\mathrm{s}}$ (+1 strange), $\overline{K^0} = \overline{\mathrm{d}}\mathrm{s}$ (-1 strange)

- ∗ K^+ = u$\bar{s}$ (+1 strange), K^- = $\bar{u}$s (-1 strange)

- Baryons are also hadrons, but consist of three quarks — all of which are antiquarks in an antibaryon.
  - proton = uud, antiproton = $\overline{u}\overline{u}\overline{d}$
  - neutron = udd
  - The proton is the only stable baryon — a free neutron decays into a proton, releasing an electron and antineutrino (β^- decay)
- Quarks are key to β decay
  - β^- decay — d→u quark (neutron to proton)
  - β^+ decay — u→d quark (proton to neutron)
- When balancing equations note that strangeness is conserved in any strong interaction
  - but in weak interactions strangeness can change by 0, +1 or -1 (because strange particles decay in the weak interaction)

2 The photoelectric effect

- If we shine light with high enough frequency on metals, photoelectrons are released.
  - no photoelectrons emitted if the incident frequency < threshold frequency, f_T
  - rate of electron emission $\propto$ intensity
- The photoelectric effect could not be explained by wave theory as this states:
  - for a certain frequency, energy $\propto$ intensity
  - energy would spready evenly across the wavefront
  - each free e$^-$ would gain some energy
  - gradually each free e$^-$ would gain enough to leave
- No explanation for E_k depending only on f, or for the existence of f_T
- could only be explained by the theory of 'packets' ie photons.

$$E = hf = \frac{hc}{\lambda}$$

$$\text{For e}^- \text{ release, } hf \geq \phi \text{ (work function) so } f_T = \frac{\phi}{h}$$

$$hf = \phi + E_k \text{ max}$$

$$\text{Stopping potential gives max } E_k\text{: } e \times V_s = E_k \text{ max}$$

2.1 Energy levels

- e$^-$ can move down an energy level by photoemission
- 'e × V = E_k carried by an electron accelerated through a 1V potential difference'
- energy gained by electron = accelerating potential difference
- energy carried by each photon is equal to the difference in energy between the two levels (E_2 = lower energy level):
$$\Delta E = E_2 - E_1 = hf$$
- In excitation electrons move up energy levels if they absorb a photon with sufficient energy to cover the difference.
- If electrons emit photons, they can move down energy levels - de-excitation.
Energy of the photon emitted = $hf = E_1 - E_2$ (E_2 lower level)

- If an electron is removed from an atom it is ionised — energy of each level in the atom is equal to the energy required to ionise from that level.
  - ground state = 'ionisation energy'

Line spectra are evidence for the transitions between discrete energy levels in atoms. If we look at a tube of glowing gas through a prism we see a spectrum of discrete lines, rather than continuous colours. The pattern of wavelengths is unique to each element. The wavelength is linked to the energy of the photons released when electrons de-excite.

2.2 The fluorescent tube

1. An initial high voltage is applied across mercury vapour. This accelerates free electrons, which ionise some of the mercury atoms, producing more free electrons.
2. Free electrons collide with electrons in other mercury atoms, exciting them to higher levels.
3. When the excited electrons return to ground states they emit UV photons.
4. Phosphor coating on the tube absorbs these particles, exciting its electrons.
5. The excited phosphorous electrons de-excite in steps, emitting lower energy visible photons.

2.3 Wave-particle duality

- Interference and diffraction show light as a wave, but the photoelectric effect shows it as a particle.
- Electron diffraction shows the wave nature of electrons
  - diffraction patterns showed when accelerated electrons in vacuo interact with the spaces in graphite crystal
  - following wave theory, the spread of the lines increased if wavelength increased. Slower electrons = wider spacing.

$$\text{de Broglie } \lambda = \frac{h}{mv}$$

- A vacuum photocell is a glass tube containing two metal plates — a photocathode and photoanode, when light of frequency ¿ f_T of the metal is incident on the photocathode, electrons are emitted from the cathode and are attracted to the anode. A microammeter can measure the photoelectric current, which is proportional to the number of electrons per second that transfer from the cathode to the anode

3 Waves

- A progressive wave carries energy from one place to another without transferring any material.
  - transverse — direction of oscillation is perpendicular to direction of energy transfer
  - longitudinal — oscillation is parallel to energy transfer
- displacement — how far a point on the wave has moved from the undisturbed position
- amplitude — maximum magnitude of displacement
- phase — measurement of the position of a certain point along the wave cycle

$$\text{phase difference in radians} = \frac{2\pi d}{\lambda} \text{ for distance } d \text{ apart}$$

- Polarised waves oscillate in only one direction
  - polarisation can only happen for transverse waves
  - a polarising filter only transmits waves in one plane
- Superposition occurs when two or more waves pass through each other — the displacements due to each wave combine.
  - Principle of superposition: *'when two or more waves cross, the resultant displacement equals the vector sum of the individual displacements'*

3.1 Interference

- Interference can be constructive or destructive — matching displacements are constructive, opposite are destructive. If the crest and trough are not of the same magnitude of displacement the destructive interference will not be total.
- For interference to occur the two waves must be coherent — having *'the same wavelength and frequency, and a fixed phase difference'*
- Interference type depends on path difference
  - Constructive interference: path difference $= n\lambda$
  - Destructive interference: path diference $= (n + \frac{1}{2})\lambda$

3.2 Stationary waves

- stationary wave = *'superposition of two progressive waves with the same frequency/wavelength'*. Unlike progressive waves no energy is transferred by a stationary wave.
- stationary waves on strings and pipes are similar:
  - at f_0: $l = \frac{\lambda}{2}$, $2f_0$: $l = \lambda$, $3f_0$: $l = \frac{3\lambda}{2}$
    where l is a fixed length of string or open pipe.
  - the distance between adjacent nodes is $\frac{\lambda}{2}$
  - a node is a point of zero displacement, an antinode a point of maximum displacement
- the longer/heavier/looser the string, the lower the resonant frequency
  μ = mass per unit length, T = tension

$$f_0 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

3.3 Interference continued

The double slit experiment gives us two coherent sources. The slits need to be about the same width as the wavelength of the radiation so that it diffracts. The result is then a pattern of light and dark fringes, where light indicates constructive interference and dark destructive interference.

$$w = \frac{\lambda D}{s}$$

w = fringe spacing, D = distance from screen, s = slit spacing.
With a non-monochromatic light source the fringes consist of a rainbow spreading out, with white at the centre.

3.4 Diffraction

- The gap needs to be similar in width to the wavelength. If it is much bigger then there will be no diffraction.
- When light passes through a single slit, we get an interference pattern. This consists of a central bright fringe, with dark and bright fringes alternating either side.
  - a graph of intensity vs distance shows that either side of the central maxima, intensity approximately halves
  - if white light is diffracted, we get colour spectra
  - increasing slit width means less diffraction — a narrower, brighter central maximum
  - increasing wavelength decreases diffraction (same slit width) — a wider, dimmer central maximum

3.4.1 Diffraction gratings

Interference patterns are sharper when we diffract through more slits — giving narrower and brighter fringes which are easier to measure. Monochromatic light on a diffraction grating gives sharp lines.

$$d \sin\theta = n\lambda \text{ where } d = \text{slit spacing, } n = \text{order (starting at 0)}$$

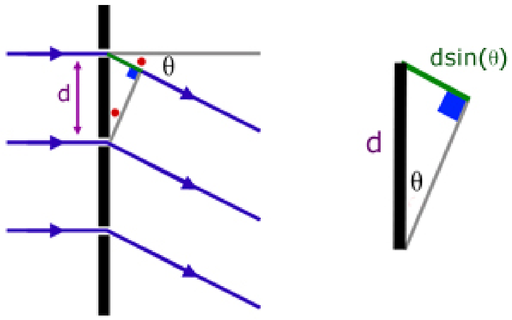

Figure 1: Diffraction grating derivation (a-levelphysicstutor.com)

3.4.2 Deriving the diffraction grating formula

1. 1^{st} order maximum happens when waves from one slit line up with waves from the next slit that are exactly λ behind.
   - the angle between 1^{st} order maximum and the incoming light is θ, path difference is λ
   - note the central maximum is the zero-order maximum
2. From the diagram, $\sin\theta = \frac{\lambda}{d}$ — hence $d \sin\theta = \lambda$
   - For the 2^{nd} order, path difference $= 2\lambda$. Hence for the n^{th} order $d \sin\theta = n\lambda$

- Increasing λ = fringes more spread out, increasing d = less spread out. $\theta < 90°$ as $\sin 90°$ is the maximum possible.
- X-ray λ similar to the atom spacing in a crystalline structure, so X-rays form a diffraction pattern when directed at thin crystal — the spacing can be found from the diffraction pattern: 'X-ray crystallography'

3.5 Refraction

- Absolute refractive index is a measure of optical density

$$n = \frac{c}{c_s}$$

- Refractive index between two media, ${}_1n_2$ is a ratio of the speed of light in material 1 to that in material 2

$${}_1n_2 = \frac{c_1}{c_2} = \frac{n_2}{n_1}$$

  We can assume n at an air—substance boundary is the absolute n of a substance.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

- When a wave passes from a dense medium into a less dense medium, it bends away from the normal as it speeds up. The reverse is true.

3.5.1 Total internal reflection

- The critical angle is the key to total internal reflection.
  - If light is incident at θ_c to the normal then the ray will exit along the flat surface
  - But if the angle of incidence is greater than θ_c, total internal reflection occurs.
- rearranging Snell's law:

$$\sin\theta_c = \frac{n_1}{n_2} =_1 n_2$$

- TIR is useful in fibre optics.
  - The core of the fibre has a high refractive index, but is surrounded by cladding of lower refractive index, which helps to protect the core from scratches (which could allow light to escape) and decreases θ_c to ensure TIR occurs.

- There are several issues encountered with fibre optics
  - Absorption — loss in amplitude as light travels along the fibre. Can be reduced by increasing purity of the glass or using repeaters at frequent intervals.
  - Modal dispersion — light enters the fibre at different angles so can take different paths through the fibre, which results in pulse broadening. Can be mitigated by using monomode fibre.
  - Material dispersion — different wavelengths of light travel at different speeds through the glass (higher n for that λ, lower the speed). Using monochromatic light sources mitigates this issue.

4 Mechanics

4.1 Vectors

- Scalars have magnitude only, whereas vectors have both magnitude and direction.
- Resolving vectors by calculation:
  - Horizontal component: $X = R\cos\theta$
  - Vertical component: $Y = R\sin\theta$
    Note θ measured from the horizontal.
- Finding the resultant vector
  - $\theta = \arctan\frac{Y}{X}$
  - $R = \sqrt{X^2 + Y^2}$
- Free-body diagrams should contain all forces acting on an object but not any forces exerted by the object itself.
- Three coplanar forces acting on a body in equilibrium will form a closed loop — triangle of forces.
- On an inclined plane the weight of the object acts straight down, but the normal reaction at a right angle to the plane. Friction acts against the object sliding down the plane. Note that the angle between mg and the normal to the plane (ie the reaction force) is the same as the slope angle.

4.2 Moments

$$\text{moment} = \text{force} \times \text{pependicular distance from the line of action of the force to the pivot, unit: N m}$$

The principle of moments says *'for a body to be in equilibrium, the sum of the clockwise moments about any point must equal the sum of the anticlockwise moments about that point'*

- A couple is a pair of coplanar forces of equal size acting parallel to each other but in opposite directions

$$\text{moment of couple} = F \times \text{distance between forces}$$

- The centre of mass of an object is the point we can consider all the weight of an object to act through. The c.o.m. is at the centre of a uniform regular solid.

4.3 Motion

- On displacement—time graphs gradient = velocity
- On velocity—time graphs gradient = acceleration & area under graph = displacement

4.3.1 Equations of uniform acceleration

$$v = u + at$$
$$s = \frac{u+v}{2}t$$
$$s = ut + \frac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$

4.3.2 Projectile motion

- In suvat equations, $a = g$. g always acts downwards ie negative.
- Horizontal and vertical components of motion must be thought of separately
  - vertical motion — constant a due to g being constant
  - horizontal motion — constant speed until projectile lands
- If a projectile is launched at an angle we need to resolve the initial velocity into vertical and horizontal components
$$X = R\cos\theta$$
$$Y = R\sin\theta$$
- resistance $\propto$ speed
- Friction/drag acts in the opposite direction to the motion of the object and converts kinetic energy to heat & sound energy. It increases with speed
- Lift acts perpendicular to fluid flow
- The terminal velocity occurs where the driving force is constant and there is a resistance force which increases with speed
  - Maximum speed is affected by the magnitude of the driving force, and the magnitude of the resistance force.

4.3.3 Newton's Laws of Motion

1. Velocity of an object will not change unless a resultant force acts upon it.
2. Acceleration of an object $\propto$ the magnitude of the resultant force
   - Force is the rate of change of momentum
   $$F = ma = m\frac{\Delta V}{\Delta t} = \frac{\Delta(mv)}{t} \text{ therefore } F\Delta t = \Delta(mv)$$
   - The impulse, defined as $F \times t$, thus equals the change in momentum. It is also the area under a force—time graph.
   - all objects fall at the same rate regardless of mass (but resistance does play a part)
3. *'If body A exerts a force on body B, then body B exerts an equal but opposite force on body A'* — aka every action has an equal & opposite reaction.

- Momentum is always conserved, assuming no external forces act.
$$\text{momentum, } p = mv$$
  - In an elastic collision, both momentum and energy are conserved.
  - In an inelastic collision, energy is not conserved.

4.4 Energy

$$\text{work done} = \text{force} \times \text{distance moved}$$

Note that force is not always in the same direction as the movement eg for a sled being pulled by a string, only the horizontal force causes the motion.

$$\text{horizontal: } W = Fs\cos\theta$$
$$\text{vertical: } W = Fs\sin\theta$$

- Area under a force—displacement graph tells us the work done

$$P = \frac{\Delta W}{\Delta t} = \frac{E}{t} = Fv$$

4.4.1 Conservation of energy

'Energy cannot be created nor destroyed, it can be transferred from one form to another, but the total energy in a closed system cannot change'

$$E_k = \frac{1}{2}mv^2$$

$$E_p = mgh$$

5 Materials

$$\text{density, } \rho = \frac{\text{mass, } m}{\text{volume, } v} \text{ unit: kg m}^{-3}$$

- Hooke's law states extension of a stretched object $\propto$ force

$$F = k\Delta l$$

  - This only applies up to the elastic limit, after which the material will be permanently stretched.
  - This plastic deformation results in a non-zero intercept on a F—Δl graph, but the gradient of such a graph remains the same as the forces between bonds are identical.

- Elastic — returns to original shape and size when force is removed
- Plastic — material is permanently stretched

5.1 Stress and strain

$$\text{stress} = \frac{F}{A} \text{ unit: Pa, N m}^2$$

$$\text{strain} = \frac{\Delta l}{l} \text{ (no units)}$$

Elastic strain energy is the area below a force—extension graph.

$$\text{Elastic strain energy, } E = \frac{1}{2}F\Delta l = \frac{1}{2}k\Delta l^2$$

The Young modulus is a property of a material — it measures stiffness.

$$\text{Young modulus, } E = \frac{\text{stress}}{\text{strain}} = \frac{Fl}{A\Delta l} \text{ unit: Pa, N m}^2 5$$

- The gradient of a stress—strain graph is thus equal to the Young modulus.
- Looking at a stress—strain graph, there are three key points: the limit of proportionality, after which the relationship is no longer linear, the elastic limit, past which plastic deformation occurs, and the yield point — after this point, the material suddenly starts to stretch without extra load.
- The stress—strain graph of a brittle material has no curve — it just stops.
- To measure the Young modulus, we need a long thin wire of the material — record the extension and the weight applied, and plot a graph. The graph can then be converted to stress—strain, or as the gradient is $\frac{\Delta l}{F}$, $\frac{1}{\text{gradient}} \times \frac{l}{A}$ = Young modulus

6 Electricity

$$Q = It \text{ so current=rate of flow of charge}$$

$$W = QV \text{ so potential difference is the energy per unit charge}$$

$V = IR$, for an Ohmic conductor $I \propto V$. A shallower gradient on I—V graph = increased resistance

- An Ohmic conductor as a straight line I—V graph

- A silicon diode conducts no current until $V \approx 0.7$ V, after which current flows with very little resistance - the graph should be an almost-vertical line.
- A filament bulb gives an S-curve: greater resistance at higher voltages as the filament heats up due to increased current flow.
- The unknown-resistor circuit consists of a variable resistor in series with the unknown resistance, an ammeter and a voltmeter in parallel with the unknown resistance. It can be used to determine the resistance of the unknown resistor.

6.1 Resistivity

$$\text{resistivity, } \rho l = RA \text{ unit: } \Omega \text{ m}$$

Superconductors have a resistivity of 0 Ω m. These are certain materials, which must be cooled below a 'transition temperature'.
Uses include power transmission lines, strong electromagnets, and very high speed electronic systems.

6.2 Power

$$P = \frac{E}{t} = IV = \frac{V^2}{R} = I^2 R$$

$$E = VIt$$

6.3 EMF and internal resistance

The internal resistance of a cell can be imagined much like a resistor in series with the cell.

$$\text{Electromotive force, } \varepsilon = \frac{\text{energy, } E}{Q} = I(R + r) = \text{terminal pd} + \text{lost volts}$$

Note that Ohm's Law still applies — $\varepsilon = Ir$
It is helpful to have awareness of potential dividers and resistive input transducers in this section.

7 Circular motion

$$\text{Angular speed, } \omega = \frac{\theta}{t} = 2\pi f \text{ unit: rad s}^-1$$

$$\text{Linear velocity, } v = \frac{2\pi r}{T} = 2\pi f r = r\omega$$

$$\text{Magnitude of centripetal accelleration is given by } a = \frac{v^2}{r}$$

$$\text{Using } F = ma,\ F = \frac{mv^2}{r} = mr\omega^2$$

7.1 Humpback bridge & 'looping the loop'

To keep a string taut, the magnitude of the centripetal force must be greater than or equal to the weight.

$$\text{so } mv^2 \geq mg \text{ or alternatively } mr\omega^2 \geq mg$$

$$\text{Tension in the string at the top } = \frac{mv^2}{r} - mg$$

$$\text{Tension in the string at the bottom } = \frac{mv^2}{r} + mg$$

Keeping a car on a humpback bridge requires weight to equal the centripetal force ie $mg \geq \frac{mv^2}{r}$.
Support force from the road $= mg - \frac{mv^2}{r}$.

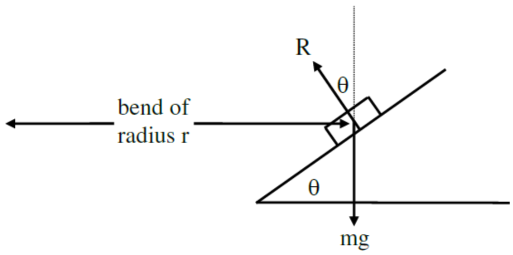

Figure 2: Motion round a banked track

7.2 Motion round a banked track

$$\text{The vertical compoment is weight: } mg = R\cos\theta$$

$$\text{The horizontal component is centripetal force: } F = \frac{mv^2}{r} = R\sin\theta$$

$$\text{For there to be no sideways friction: } v = \sqrt{gr\tan\theta}$$

8 Simple harmonic motion

$$\text{Phase difference in radians } = 2\pi\frac{\Delta t}{T}$$

Δt is the time between successive instants where the two objects are at maximum displacement in the same direction.

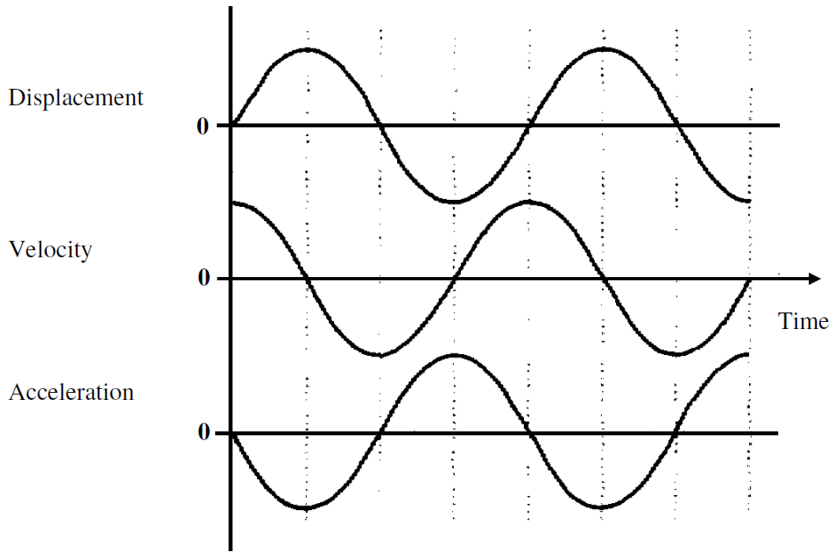

Figure 3: For a body executing SHM, these graphs are true.

- SHM is oscillating motion where the acceleration is:
  - proportional to displacement
  - in the opposite direction to displacement
- Thus the graphs of displacement and acceleration are in antiphase.
- The definition of SHM leads to $a = -\omega^2 x$ (a=amplitude, x=displacement)
  - General solution: $x = A\sin(\omega t + \phi)$ where ϕ is phase difference between t=0 and x=0
  - If timing starts at the centre ie $x = 0$, $x = A\sin(\omega t)$
  - whereas if timing starts at $x = +A$, $x = A\cos(\omega t)$ works.

$$v = \pm\omega\sqrt{A^2 - x^2}$$

8.1 Mass—spring system

$$T = 2\pi\sqrt{\frac{m}{k}}$$

T is increased by adding mass or using a weaker spring. Note that it does not depend on g.

8.2 Simple pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

T is increased by increasing the length of the pendulum. Note the 'small angle approximation' — the angle of swing must be less than 10°.

8.3 Variation of energy with displacement

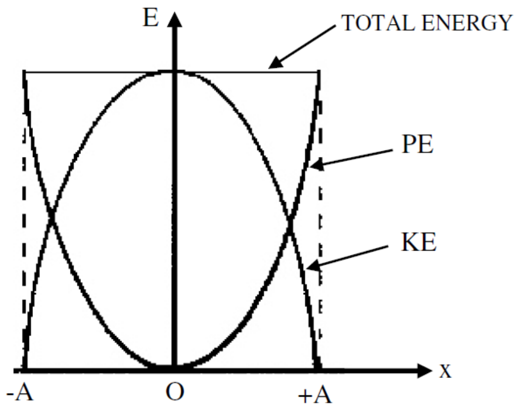

Figure 4: Variation of energy with displacement

$$E_p = \frac{1}{2}kx^2$$

$$E_k = \frac{1}{2}k(A^2 - x^2)$$

$$E_total = \frac{1}{2}kA^2$$

8.4 Damping

- Light damping — T independent of amplitude so T remains constant as amplitude decreases. Amplitude gradually decreases by the same fraction each cycle.
- Critical damping — the system returns to equilibrium in the shortest possible time without overshooting
- Heavy damping — so strong that the displaced object returns to equilibrium much more slowly than if the system is critically damped — no oscillation occurs.

8.5 Forced vibrations and resonance

When a system oscillates without a periodic (driving) force applied, it oscillates at its natural frequency. Forced vibrations occur when a periodic force is applied to a system.

- As the applied frequency increases from 0:
  - amplitude of oscillation increases until a maximum is reached at a particular frequency — this is the resonant frequency — and the amplitude then decreases again.

- phase difference between the displacement and the periodic force increases from 0 to $\frac{\pi}{2}$ at the maximum amplitude, and then from $\frac{\pi}{2}$ to π as frequency increases further.

When the system oscillates with maximum amplitude the phase difference between the displacement and the periodic force is $\frac{\pi}{2}$. The periodic force is then exactly in phase with the velocity of the system and resonance occurs.

- The lighter the damping:
  - the greater the maximum amplitude at resonance
  - the closer the resonant frequency to the natural frequency
  - hence the peak on a resonance curve will be much sharper with lighter damping.
- As the applied frequency becomes much larger than the resonant frequency:
  - amplitude of oscillations decreases more and more
  - phase difference between displacement and periodic force increases from $\frac{\pi}{2}$ until the displacement is π out of phase with the force.
- For an oscillating system with little to no damping, at resonance the applied frequency of the periodic force = the natural frequency of the system.

9 Gravitational fields

A force field is a region in which a body experiences a non-contact force. A force field can be represented as a vector, the direction of which must be determined by inspection.

- Gravity is a universal attractive force which acts between all matter.

$$\text{magnitude of a force between point masses, } F = \frac{Gm_1m_2}{r^2} \text{ where } G \text{ is the gravitational constant}$$

- A gravitational field can be represented by field lines — also known as lines of force. This is the path followed by a small mass placed close to a massive body.
  - Note that for a radial field, the field lines point towards the centre. In a uniform field eg close to the Earth's surface, field lines act straight down — parallel to each other and evenly spaced.
- The gravitational field strength, g, is the force per unit mass on a small test mass placed in the field. $g = \frac{F}{m}$
- In a radial field, the magnitude of $g = \frac{GM}{r^2}$

9.1 Gravitational potential

- Gravitational potential at a point is the gravitational potential energy per unit mass of a small test mass.
  - This is equal to the *work done per unit mass to move an object from infinity (where potential = 0) to that point.*

$$\text{gravitational potential, } V = \frac{W}{m} \text{ unit: J kg}^{-}1$$

$$\text{work done moving mass } m\text{: } \Delta W = m\Delta V$$

$$\text{gravitational potential in a radial field: } V = -\frac{GM}{r}$$

  - The negative sign is due to the reference point being infinity, and the fact that other than at infinity the force is in fact attractive.
- ΔV can be found from the area of a g—r graph
- Equipotentials are surfaces of constant potential — no work needs to be done to move along an equipotential surface.
- Potential gradient at a point in a gravitational field is the change of potential per metre at that point

- In general, for ΔV over a small distance Δr, potential gradient $= \frac{\Delta V}{\Delta r}$
- Gravitational field strength is the negative of potential gradient:

$$g = -\frac{\Delta V}{\Delta r}$$

  the minus sign shows g acts in the opposite direction to potential gradient.
- g—r graph for a planet of radius R shows up to R, g increases at a constant rate. However, outside R it follows an inverse-square law — at $2R$, g is $\frac{1}{4}$ of the value at the surface.
- Gravitational potential is similar, but starts only at R. The graph goes from a negative value (potential at surface) and tends towards zero, at a non-linear rate.

9.2 Orbits and satellites

If an object is moving parallel to a planet's surface at the correct speed such that the centripetal force required is matched exactly by the force of gravity, it will orbit.

For a satellite orbiting at distance r from the centre of a planet: $\dfrac{GM_{planet}m}{r^2} = \dfrac{mv^2}{r} = mr\omega^2$ showing m irrelevant

- For geostationary orbit, $T_{sat} = T_{planet}$, so for earth $T \approx 86\ 400$ s.
  - $T = \frac{2\pi}{\omega}$

9.2.1 Kepler's 3rd Law

For an object in orbit around mass M:

1. $\frac{GM}{r^2} = r\omega^2$ so $\frac{GM}{r^3} = \omega^2$
2. Combining with $T = \frac{2\pi}{\omega}$ gives $\frac{GM}{r^3} = \frac{4\pi^2}{T^2}$, or $T^2 = \frac{4\pi^2}{GM}r^3$
3. Everything is constant except T and r, meaning $T^2 \propto r^3$ — Kepler's 3rd Law
4. To further prove K3L, if $T^2 = \frac{4\pi^2}{GM}r^3$, taking logarithms gives $\log(T^2) = \log(\frac{4\pi^2}{GM}r^3)$
5. $\log(T^2) = \log(\frac{4\pi^2}{GM}) + \log(r^3)$
6. $2\log(T^2) = 3\log(r) + \log(\frac{4\pi^2}{GM})$
7. $\log(T^2) = 1.5\log(r) + 0.5\log(\frac{4\pi^2}{GM})$
8. so $\log(T^2) = 1.5\log(r) + \log(\sqrt{\frac{4\pi^2}{GM}})$
9. Hence a graph of $\log T$ against $\log r$ has gradient 1.5 and positive y-intercept of $\frac{2\pi}{\sqrt{GM}}$

9.2.2 Escape velocity

For an object to go into orbit once launched rather than fall back to Earth, it must never run out of kinetic energy. So supplied $\frac{E_k}{m} \geq V$.
Equating E_k and $V \cdot m$ allows us to work out that

$$\text{escape velocity, } v = \sqrt{\frac{2GM}{r}}$$

9.2.3 Energy considerations

A satellite has $E_k = \frac{1}{2}mv^2$. Equating forces in orbit gives $\frac{mv^2}{r} = \frac{GMm}{r^2}$ or $v^2 = \frac{GM}{r}$.

$$\text{Hence to be in orbit, } E_k = \frac{GMm}{2r}$$

$$\text{Potential energy is calculated from gravitational potential: } E_p = -\frac{GM}{r} \cdot m$$

$$\text{The total energy is the sum: } E_T = \frac{GMm}{2r} + (-\frac{GMm}{r})$$

$$E_T = -\frac{GMm}{2r}$$

10 Electrostatics

$$\text{Force between point charges in vacuo, } F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2}$$

- ε_0 = permittivity of free space
  - air can be treated as a vacuum when calculating force between charges
  - for a charged sphere, charge may be considered to be concentrated at the centre
- Electric fields can be represented by field lines — the direction of which is positive to less positive.
  - An electric line of force is the path along which a free positive charge would tend to move.
- Electric field strength at a point in an electric field is the force exerted by the field by a unit positive charge placed at that point

$$\text{electric field strength, } E = \frac{F}{Q} \text{ unit: N C}^{-1} \text{ or V m}^{-1}$$

Therefore the force exerted on charge Q at a point is given by $F = EQ$

$$\text{magnitude of field strength in a \underline{uniform} field, } E = \frac{V}{d}$$

  - This can be derived from the work done moving a charge between the plates: $Fd = Q\Delta V$

$$\text{field strength in a \underline{radial} field, } E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

10.1 Electric potential

The electric potential at a certain position in any electric field is the *'work done per unit positive charge on a positive test charge when it is moved from infinity to that position'*. Hence electric potential = 0 at infinity.

$$\text{Electric potential, } V = \frac{\text{work done, } W}{Q} \text{ unit: J C}^{-}1$$

$$\text{Work done moving charge } Q,\ \Delta W = Q\Delta V$$

$$\text{magnitude of electric potential in a \underline{radial} field, } V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Unlike gravitational potential, electric potential is a <u>scalar</u> quantity.

The electric potential of a positively charged particle increases as it moves to a point at higher potential — it gains energy from work having to be done to move it against electrostatic repulsion.

Potential difference between two points in an electric field is equal tot eh work done in moving a unit positive charge from the point at lower potential to the point at higher potential.

- The <u>potential gradient</u> at any position in an electric field is the change in potential per unit change of distance in a given direction.

$$\text{electric field strength, } E = -\text{potential gradient} = -\frac{\Delta V}{\Delta r}$$

10.1.1 Graphical representations of E and V with r

- E—r graph follows an inverse-square law as $E \propto \frac{1}{r^2}$, but there is no electric field strength inside the charged sphere itself.
  - Hence graph starts at r rather than 0, and rapidly approaches 0.
  - ΔV can be found from the area under this graph as $E = -\frac{\Delta V}{\Delta r}$
- V—r graph is constant from 0 to r, then falls at a rate lesser than E—r graph as $V \propto \frac{1}{r}$

10.1.2 Projectile movement

A charged particle aimed through a uniform field will accelerate in one plane only, resulting in a parabolic arc similar to a ball thrown horizontally on Earth.

Relative strength: electric forces in a hydrogen atom are approximately 10^{39} times stronger than the gravitational forces acting.

11 Capacitance

A capacitor is any device used to store charge. The capacitance of an isolated conductor is the ratio of charge stored to the change in electric potential.

$$\text{capacitance, } C = \frac{Q}{V} \text{ unit: Farad, F}$$

$$\text{For a parallel plate capacitor, } C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

11.1 Energy stored

Charging a capacitor means transferring charge from the plate at lower potential to the plate at higher potential, which requires energy. Thus work done in charging = energy stored.

If a capacitor is charged to V by Q then the area under a V—Q graph gives the work done.

$$\text{work done, } W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$

11.2 Discharging

$$\text{Charge left on a capacitor } t \text{ s after it starts discharging, } Q = Q_0 e^{-\frac{t}{RC}}$$

For a discharging capacitor the graphs of charge, voltage and current against time all have the same shape, so this formula works for V and I too.

The time constant is t taken for Q to fall to $\frac{1}{e}$ of its previous value. $T = RC$

From this, we can calculate that the time for charge or voltage to half in value is $0.693RC$.

11.3 Charging

The rate of charge leaving from or arriving on a capacitor depends on how much charge is already there. More work needs to be done to push electrons onto a partially charged capacitor than an empty one.

$$\text{For a charging capacitor, } Q = Q_0(1 - e^{-\frac{t}{RC}})$$

The graphs of Q and V against t show that charge & voltage increase rapidly at first, but the rate of change decreases as a maximum is approached. This means this equation works for V as well as Q, but not I (which looks the same for both a charging and discharging capacitor).

Increasing R leads to a shallower charging or discharging curve which takes longer to reach its maximum or minimum. R decreases the current — decreasing the rate of flow of charge.

$$I = \frac{Q}{t}$$

11.4 Polarised molecules

Some molecules have one part more positive and another more negative — they are polarised.

If a polarised molecule is placed in an electric field, the two ends respond differently to the field, moving in opposite directions, rotating the molecule until it lines up with the field.

12 Electromagnetism

12.1 Magnetic flux density

Force on a current-carrying wire when field is perpendicular to current, $F = BIl$

- Fleming's left hand rule can be used to calculate the direction in which the wire will move.
  - thumb — thrust
  - first finger — field
  - second finger — current

The strength of a magnetic field is given by its flux density, B, which is measured in Tesla, T.

Flux density is a vector — its direction is along a tangent to the field line at that point. Its magnitude is represented by the density of magnetic field lines.

One Tesla is defined as *'the flux density of the field that produces a force of 1 N on a unit length of conductor carrying a current of 1 A perpendicular to the field.'*

- Note that for a magnetic field, field lines always run North—South
- The right hand grip rule gives the direction of field lines where the thumb points in the direction of current flow
  - $\otimes$ represents current flowing into the page, and $\odot$ current flowing out of the page. Imagine a dart flying.

Magnetic flux passing through area A perpendicular to a field, $\phi = BA$ unit: Webers, Wb

If area is not perpendicular to the field then $\phi = BA\cos\theta$ where θ is the angle to the normal to the area, or $\phi = BA\sin\theta$ where θ is to the plane of the area. Consider the graphs and where you would expect a maximum to occur.

magnetic flux linkage $= N\phi$ where N is the number of turns cutting the flux

12.2 Charges in a magnetic field

Force on charged particles moving in a magnetic field, $F = BQv$ when field perpendicular to v

Charged particles in a magnetic field follow a circular path. The direction of the force on a positive charge is given by Fleming's LHR, and the force = centripetal force required to maintain this motion.

$$\text{so for a charged particle in a magnetic field, } BQv = \frac{mv^2}{r} = mr\omega^2$$

The cyclotron is an application of this phenomenon (magnetic deflection). Two D-shaped electrodes are separated by a small gap in an evacuated chamber placed in the uniform magnetic field of a large electromagnet.

Charged particles produced by an ion source at the centre enter one D and move in a circular path due to the field. A high-frequency alternating current is connected between the Ds, with frequency such that its polarity reverses at the same rate as the particles cross from one D to another. The energy is of the particle is increases every time it crosses from one D to the other, the radius of orbit increases as energy increases and the beam finally emerges tangentially from the cyclotron.

At radius r, magnetic force = centripetal force, so $r \propto v$

12.3 Electromagnetic induction

When a wire cuts through magnetic field lines, an electromotive force is induced in the wire.

12.3.1 Lenz's Law

'The direction of the induced current is such that it opposes the motion producing it.'

12.3.2 Faraday's Law

'The magnitude of the induced emf is proportional to the rate of change of flux linked with that circuit, or the rate at which magnetic flux is cut'

$$\text{Combined with Lenz's Law: } \varepsilon_{ind} = -\frac{\Delta}{\Delta t}(\text{flux linkage})$$

12.3.3 Flux linkage

$$\text{As flux linkage} = N\phi,\ \varepsilon_{ind} = \frac{\Delta}{\Delta t}(N\phi)$$

Given $\phi = BA$, the flux linking a coil $= BAN$ and:

$$\varepsilon_{ind} = \frac{\Delta}{\Delta t}(BAN)$$

$$\text{or average } \varepsilon_{ind} = \frac{\text{change in } BAN}{\text{time taken}}$$

Both of these assume the plane of the coil is perpendicular to the coil. Otherwise flux linking coil $= BAN\cos\theta$ or $BAN\sin\theta$ (consider maxima)

12.3.4 emf induced in a moving conductor

$$\varepsilon_{ind} = Blv$$

Fleming's right hand rule gives the direction of the induced current if a complete circuit. If asked to label emf consider the conductor just as any other source — in a wire connected between the terminals, current would flow from positive to negative.

12.3.5 emf induced in a rotating coil

For θ between the normal to the coil and the field, flux linking the coil is given by $\phi = BAN\cos\theta$. For a constant rate of rotation $\theta = \omega t$ where ω is the angular speed in rad s^{-1}.

Therefore $\phi = BAN\cos(\omega t)$.

$$\text{Combining with Faraday's Law, } \varepsilon_{ind} = BAN\omega\sin(\omega t)$$

The induced emf is therefore a sine wave with peak value $BAN\omega$. The faster the coil is rotated, the greater the peak. This very much depends on when timing starts however, so consider maxima.

These revision notes are incomplete, and are not a substitute for your own knowledge.